

INTERNATIONAL BACCALAUREATE
Mathematics: applications and interpretation
MAI

EXERCISES [MAI 2.7-2.8]
COMPOSITION – INVERSE FUNCTIONS

Compiled by Christos Nikolaidis

A. Paper 1 questions (SHORT)

1. [Maximum mark: 10]

Let $f(x) = 10 - 2x$ and $g(x) = 5x$. Calculate

- (a) $(f \circ g)(x)$ and $(g \circ f)(x)$ [2]
- (b) $(f \circ f)(x)$ and $(g \circ g)(x)$ [2]
- (c) $f^{-1}(x)$ [2]
- (d) $g^{-1}(10)$ [2]
- (e) $(f^{-1} \circ g)(x)$ and $(g \circ f)^{-1}(x)$ [2]

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2. [Maximum mark: 8]

Let $f(x) = 2x+5$ and $g(x) = 5x+3$.

(a) Calculate $(f \circ g)(1)$ by two different methods:

(i) find $(f \circ g)(x)$ first and then $(f \circ g)(1)$

(ii) directly by using the definition $(f \circ g)(1) = f(g(1))$ [3]

(b) Calculate $f^{-1}(25)$ by two different methods:

(i) find $f^{-1}(x)$ first and then $f^{-1}(25)$

(ii) directly by using the definition $f^{-1}(25) = x \Leftrightarrow f(x) = 25$ [3]

(c) Calculate $(g \circ f)(1)$ and $g^{-1}(53)$ [by any method you like!] [2]

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3. [Maximum mark: 8]

The tables below show some values of two functions f and g

x	1	2	3	4
$f(x)$	2	-3	-1	3

x	1	2	3	4
$g(x)$	5	3	1	-3

- (a) Write down the values of $g(3)$, $f^{-1}(3)$ [2]
- (b) Calculate $(f \circ g)(2)$ [2]
- (c) Calculate $(g \circ g)(3)$ [2]
- (d) Find a solution of the equation $(g \circ f)(x) = 3$ [2]

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4. [Maximum mark: 6]

The functions f and g are both defined in the interval $[-4,4]$ and g is invertible.

Some values of the functions are given below.

x	1	3	4
$f(x)$	3	2	1

x	1	3	4
$g(x)$	4	1	-3

- (a) Calculate $(g^{-1} \circ f)(4)$ [2]
- (b) Find a solution of the equation $(f \circ g)(x) = 1$ [3]
- (c) Write down the value of $(g^{-1} \circ g)(2)$ [1]

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5. [Maximum mark: 6]

Let $f(x) = 2x + 1$ and $g(x) = 3x^2 - 4$. Find

- (a) $f^{-1}(x)$; [2]
- (b) $(g \circ f)(-2)$; [2]
- (c) $(f \circ g)(x)$. [2]

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6. [Maximum mark: 4]

The function f is defined by $f: x \mapsto \sqrt{3-2x}$, $x \leq \frac{3}{2}$

Evaluate $f^{-1}(5)$.

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7. [Maximum mark: 6]

Let $g(x) = 3x - 2$, $h(x) = \frac{5x}{x-4}$, $x \neq 4$.

- (a) Find an expression for $(h \circ g)(x)$. Simplify your answer. [3]
- (b) Solve the equation $(h \circ g)(x) = 0$. [3]

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8. [Maximum mark: 6]

The functions f and g are defined by $f: x \mapsto 3x$, $g: x \mapsto x + 2$.

- (a) Find an expression for $(f \circ g)(x)$. [2]
- (b) Show that $f^{-1}(18) + g^{-1}(18) = 22$. [4]

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9. [Maximum mark: 5]

Let $f(x) = x^2$ and $g(x) = 2x - 3$.

(a) Find $g^{-1}(x)$. [2]

(b) Find $(f \circ g)(4)$. [3]

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10. [Maximum mark: 5]

Let $f(x) = 7-2x$ and $g(x) = x+3$.

(a) Find $(g \circ f)(x)$. [2]

(b) Write down $g^{-1}(x)$. [1]

(c) Find $(f \circ g^{-1})(5)$. [2]

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11. [Maximum mark: 6]

Let $f(x) = \sqrt{x+4}$ $x \geq -4$ and $g(x) = x^2$, $x \in \mathbb{R}$.

(a) Find $(g \circ f)(3)$ [2]

(b) Find $f^{-1}(x)$ [3]

(c) Write down the domain of f^{-1} . [1]

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12. [Maximum mark: 5]

Let $f(x) = x^3 - 4$ and $g(x) = 2x$.

(a) Find $(g \circ f)(-2)$. [2]

(b) Find $f^{-1}(x)$. [3]

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13. [Maximum mark: 4]

Two functions f, g are defined as follows:

$$f: x \mapsto 3x + 5 \qquad g: x \mapsto 2(1 - x)$$

Find

(a) $f^{-1}(2)$; [2]

(b) $(g \circ f)(-4)$. [2]

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14. [Maximum mark: 6]

Let $f(x) = 2^x$, and $g(x) = \frac{x}{x-2}$, ($x \neq 2$). Find

(a) $(g \circ f)(3)$; [3]

(b) $g^{-1}(5)$. [3]

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15. [Maximum mark: 7]

(a) Find the inverse of the function $f(x) = \frac{x}{x+5}$. [5]

(b) Given that $S = \frac{R}{R+5}$, express R in terms of S . [2]

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16. [Maximum mark: 6]

Consider the functions $f(x) = 2x$ and $g(x) = \frac{1}{x-3}$, $x \neq 3$.

(a) Calculate $(f \circ g)(x)$. [2]

(b) Calculate $(f \circ g)(4)$. [2]

Let $P = 2R$, and $R = \frac{1}{S-3}$.

(c) Express P in terms of S . [1]

(d) Given that $S = 4$, find the value of P . [1]

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17. [Maximum mark: 8]

Let $f(x) = \frac{2x-3}{3x-2}$. Show that

(a) Show that $f^{-1} = f$ (i.e. f is self-inverse) [4]

(b) Verify that $(f \circ f)(x) = x$ [4]

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18. [Maximum mark: 6]

Consider the functions $f: x \mapsto 4(x - 1)$ and $g: x \mapsto \frac{6-x}{2}$.

(a) Find g^{-1} . [2]

(b) Solve the equation $(f \circ g^{-1})(x) = 4$. [4]

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19. [Maximum mark: 4]

(a) Given functions $f: x \mapsto x + 1$ and $g: x \mapsto x^3$, find the function $(f \circ g)^{-1}$.

(b) Given $A = B + 1$ and $B = C^3$, describe what $(f \circ g)^{-1}$ represents.

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20. [Maximum mark: 10]

Let $f(x) = \frac{4}{x+2}$, $x \neq -2$ and $g(x) = x - 1$. If $h = f \circ g$, find

(a) $h(x)$ [2]

(b) $h^{-1}(x)$ [4]

Let

$K = \frac{4}{L+2}$ is the expression of K in terms of L

$L = M - 1$, is the expression of L in terms of M

(c) Describe what the following represent

- (i) $h(x)$ (ii) $h^{-1}(x)$ (iii) $h(3)$ (iv) $h^{-1}(0.5)$ [4]

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21. [Maximum mark: 7]

Consider the functions f and g where $f(x) = 3x - 5$ and $g(x) = x - 2$.

Solve the equation $(f^{-1} \circ g)(x) = (g^{-1} \circ f)(x)$.

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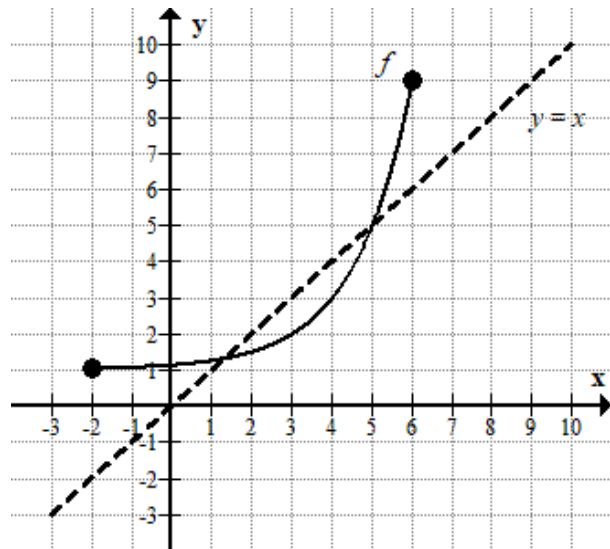
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22. [Maximum mark: 6]

The graph of the function f is shown below. On the same diagram sketch the graph of the function f^{-1} .



23. [Maximum mark: 9]

The function f is given by $f(x) = x^2 - 6x + 13$, for $x \geq 3$.

- (a) Show that f may also be written in the form $f(x) = (x - 3)^2 + 4$. [2]
- (b) **Hence** find the inverse function f^{-1} . [3]
- (c) State the domain and the range of f . [2]
- (d) State the domain and the range of f^{-1} . [2]

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24. [Maximum mark: 8]

The function f is defined as $f(x) = \frac{3x-4}{x+2}$, $x \neq -2$

(a) Find an expression for $f^{-1}(x)$. [5]

(b) Write down the domain of f^{-1} . [1]

(c) **Hence**, given that $F = \frac{2D+4}{3-D}$, express D in terms of F . [2]

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GIVEN $f \circ g$ AND ONE OF THE FUNCTIONS FIND THE OTHER ONE

25. [Maximum mark: 10]

(a) Let $f(x) = 2x + 5$ and $g(x) = 5x + 3$. Show that $(f \circ g)(x) = 10x + 11$ [2]

(b) Let $(f \circ g)(x) = 10x + 11$ and $f(x) = 2x + 5$. By using the fact

$$g = f^{-1} \circ (f \circ g)$$

find an expression for the function g . [4]

(c) Let $(f \circ g)(x) = 10x + 11$ and $g(x) = 5x + 3$. By using the fact

$$f = (f \circ g) \circ g^{-1}$$

find an expression for the function f . [4]

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26. [Maximum mark: 6]

- (a) Given that $f \circ g = h$, express f in terms of g and h . [2]
- (b) Given that $f \circ g = h$, express g in terms of f and h . [2]
- (c) Given that $f \circ g \circ h = k$, express g in terms of f , h and k . [2]

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27. [Maximum mark: 8]

The function f is defined by $f: x \mapsto x^3$.

- (a) Find $f^{-1}(x)$. [2]
- (b) Given that $(f \circ g)(x) = x + 1$, find $g(x)$. [3]
- (c) Given that $(g \circ f)(x) = x + 1$, find $g(x)$. [3]

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28. [Maximum mark: 8]

The function f is defined by $f: x \mapsto x^3 - 1$

Find an expression for $g(x)$ in terms of x in each of the following cases

(a) $(f \circ g)(x) = 2x + 1$;

[4]

(b) $(g \circ f)(x) = 2x + 1$.

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B. Paper 2 questions (LONG)

30. [Maximum mark: 12]

The function f is defined for $x \geq 0$ by $f(x) = \frac{x^2 - 1}{x^2 + 1}$.

(a) Find an expression for $f^{-1}(x)$. [6]

(b) **Hence** write down the inverse function of $g(x) = \frac{x^2 - 1}{x^2 + 1}$, for $x \leq 0$. [1]

(c) Given that $T = \sqrt{\frac{1+S}{1-S}}$, express S in terms of T . [2]

(d) Find the value of S when $T = 1.5$ [2]

(e) Write down the minimum value of S . [1]

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31. [Maximum mark: 11]

Let $f(x) = \sqrt{x+1} + 1$ and $g(x) = x^2$.

- (a) Solve the equation $(g \circ f)(x) = 1$ [3]
- (b) Find the function $h(x)$, given that $h \circ f = g$ [4]
- (c) Find the function $k(x)$, given that $f \circ k = g$ [4]

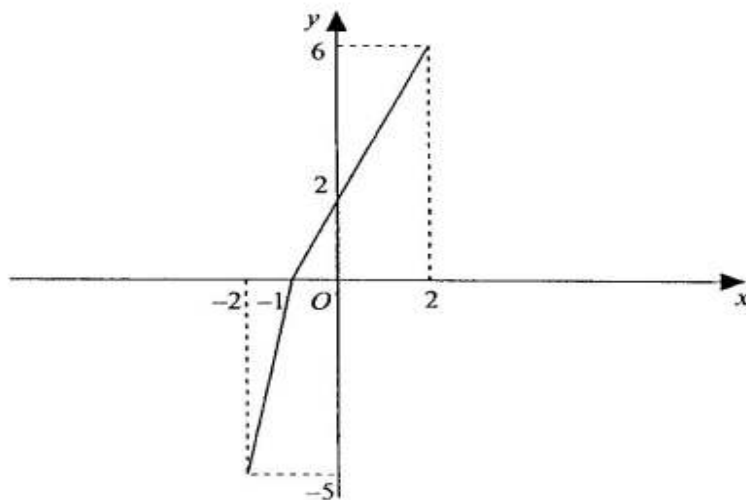
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32. [Maximum mark: 16]

The function $f(x)$ is defined by $f(x) = \frac{3x-1}{x-3}$, $x \neq 3$.

- (a) Show that f is a self-inverse function. [4]
- (b) **Hence** find, in terms of k , the result of $(f \circ f)(k)$, where $k \neq 3$ [1]
- (c) Given that $P = \frac{3Q-1}{Q-3}$, express Q in terms of P . [2]

The figure below shows a sketch of a one-to-one function $g(x)$ defined over the domain $-2 \leq x \leq 2$. The graph of $y = g(x)$ consists of two straight line segments and the range of $g(x)$ is $-5 \leq g(x) \leq 6$.



- (d) Find the values of
 - (i) $(f \circ g)(-2)$ (ii) $(g \circ f)(-5)$ (iii) $(g \circ g)(0)$. [6]
- (e) On the same diagram above, sketch the graph of the inverse function $y = g^{-1}(x)$ and state its domain. [3]

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A series of 30 horizontal dotted lines providing space for student work.

